TRINITY COLLEGE

Mathematics Applications 3,4
Test 6 2019

Calculator Assumed

Networks, Project Networks and Allocation Problems

STUDENT'S NAME SOLUTIONS

DATE: Friday 6th September **TIME:** 50 minutes **MARKS:** 50

INSTRUCTIONS:

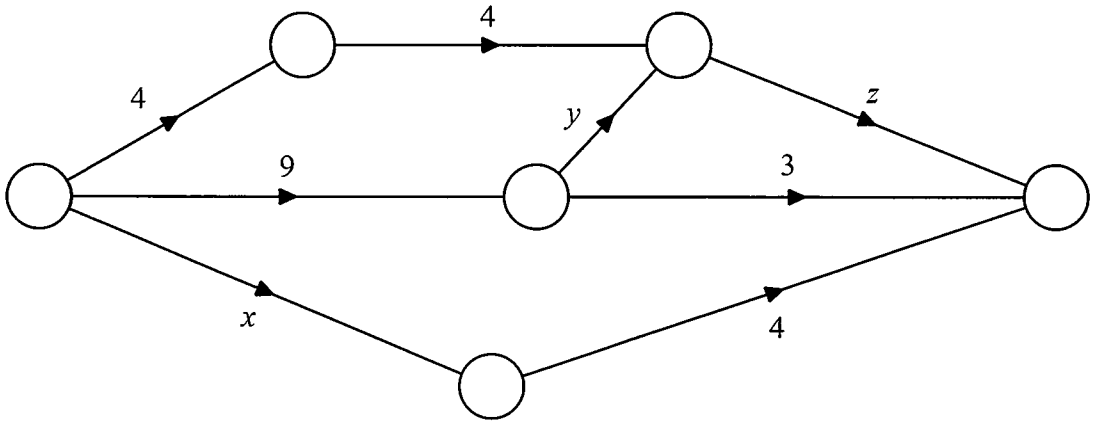
Standard Items:	Pens, pencils, drawing templates, eraser
Special Items:	Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

---

1. (3 marks)

The following graph shows the maximum flow through each path of a network. Determine the minimum values of x, y, and z that keep the network flowing at a maximum.



$x = 4$

$y = 6$

$z = 10$

2. (6 marks)

The table below gives the flying distance, in kilometres, between the towns and cities labelled A to H.

	A	B	C	D	E	F	G	H
A	-	390	260	170	235	230	270	480
B	390	-	500	670	590	290	370	210
C	260	500	-	450	490	180	190	470
D	170	670	450		250	320	280	420
E	235	590	490	250	-	270	460	720
F	230	290	180	320	270	-	350	600
G	270	370	190	280	460	350	-	340
H	480	210	470	420	720	600	340	-

The flight network is to be constructed so that it will enable a passenger to travel by air from one of these towns or cities to any other in the network, maybe not directly, but at least via other towns or cities in the network.

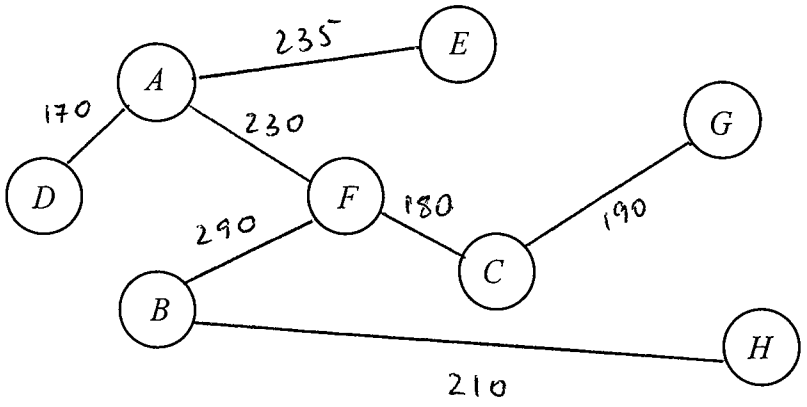
(a) Determine the minimum flying distance to connect all the towns or cities. [3]

1505 km

(b) Which towns are directly connected to F. [1]

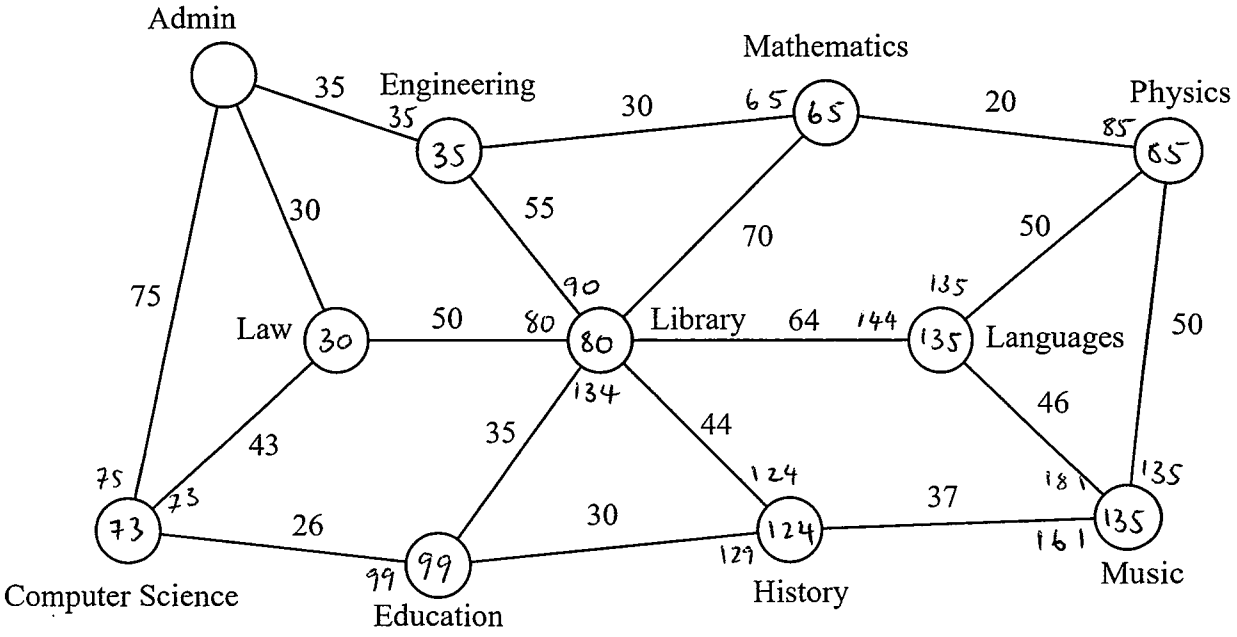
A, B + C

(c) Draw the minimum flying network below. [2]



3. (7 marks)

The network below shows the connecting paths, in metres, between buildings on a university campus.



Determine the minimum distances between the following buildings, stating the required path.

(a) The Admin building to the Music Building. [3]

working: ✓

Ad, Eng, Mat, Phys, Music ✓ 135 m ✓

(b) The Admin building to the Music building if the Languages building must be on the route taken. [2]

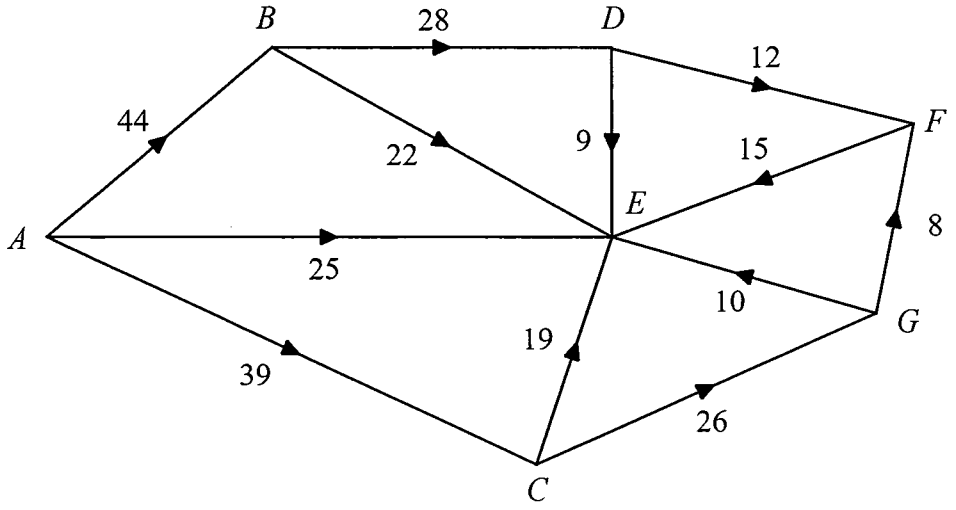
Ad, Eng, Mat, Phys, lang, Music ✓ 181 m ✓

(c) The Admin building to the Music building if the Library must be on the route taken. [2]

Ad, Law, Lib, Hist, Music ✓ 161 m ✓

4. (10 marks)

In the following network the numbers show the maximum flow of cars, in thousands, along each arc over a one-hour time frame.



(a) Clearly identify the sink. [1]

E

(b) By listing your paths, determine the maximum flow possible for one hour. [4]

ABE = 22

ABDE = 9

ABDFE = 12

AE = 25

ACE = 19

ACGE = 10

ACGFE = 3

100 ✓

✓ 4 paths correct

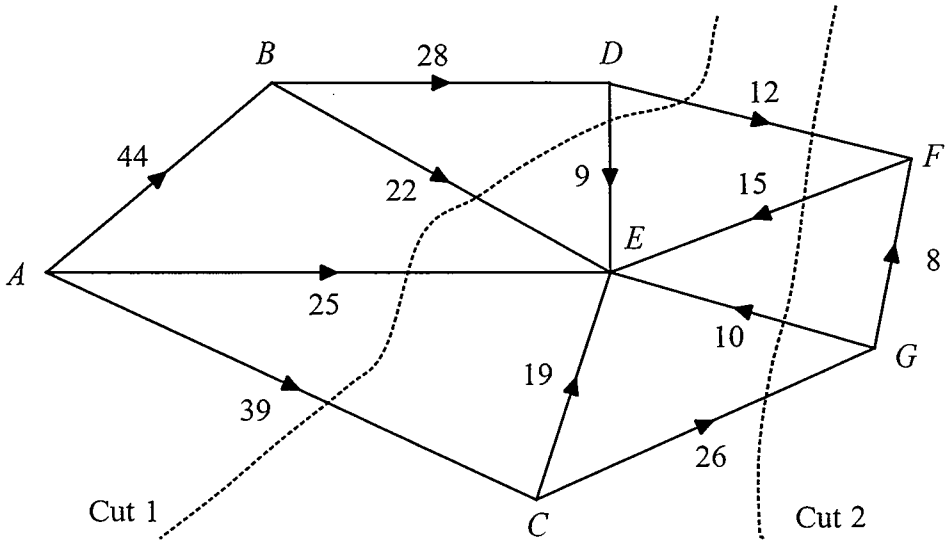
✓ All paths correct.

100 000 cars/hour ✓

(c) One path is to be upgraded. Identify which one should be upgraded and the new flow of the path to maximise the overall flow of the network. [3]

AE ✓ It can be upgraded infinitely. ✓✓

(d) Calculate the value of the cuts indicated on the network below [2]

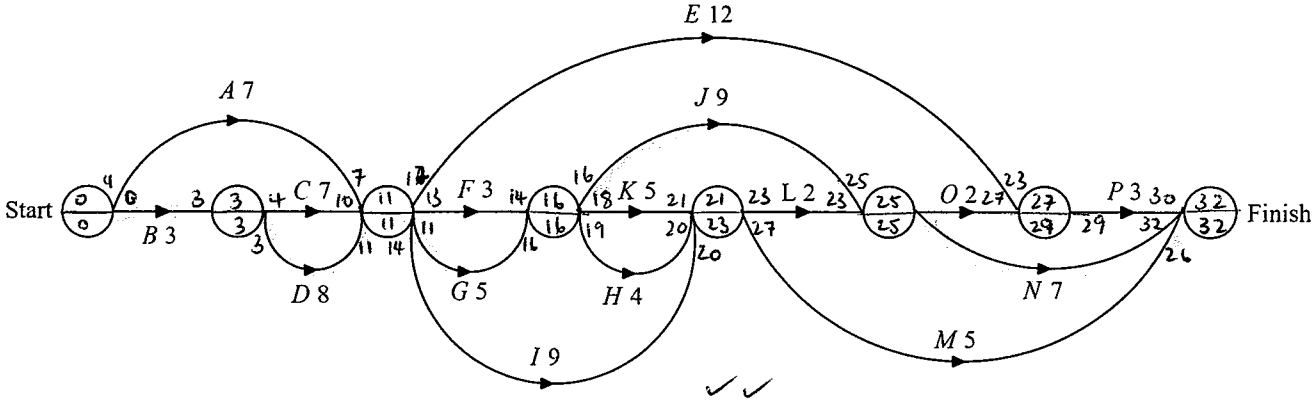


Cut 1 = 107 ✓

Cut 2 = 0 → Not a valid cut. ✓

5. (~~14~~ 13 marks)

The diagram below represents a project network. The times marked are in hours.



(a) Determine the critical path? [3]

B, D, G, J, N ✓

(b) What is the duration of the critical path? [1]

32 hours ✓

(c) Which tasks are immediate predecessors of task L? [2]

K, H & J ✓✓

The project commenced at 8 am Monday. Assuming an 8-hour day is worked, no breaks are taken and work commences at 8 am every day:

(d) (i) What is the earliest time and day that activity K could start? [2]

Mon + 2 days

8 am ✓ Wed ✓

(ii) What is the latest time and day that activity K could start? [2]

10 am ✓ Wed ✓

(e) Determine the amount of float time for activity H. [1]

3 hours ✓

(f) What would be the effect on the minimum completion time of shortening activity J by 3 hours? [1]

2 hour increase. (34 hours). ✓

(g) What would be the effect on the minimum completion time of lengthening activity I by 3 hours? [1]

No effect (32 hours) ✓

6. (4 marks)

The wet areas of an old house are to be refurbished before the new owners advertise the property for rental accommodation. To complete the refurbishment the tasks A to N must be completed with estimated times indicated in the order shown.

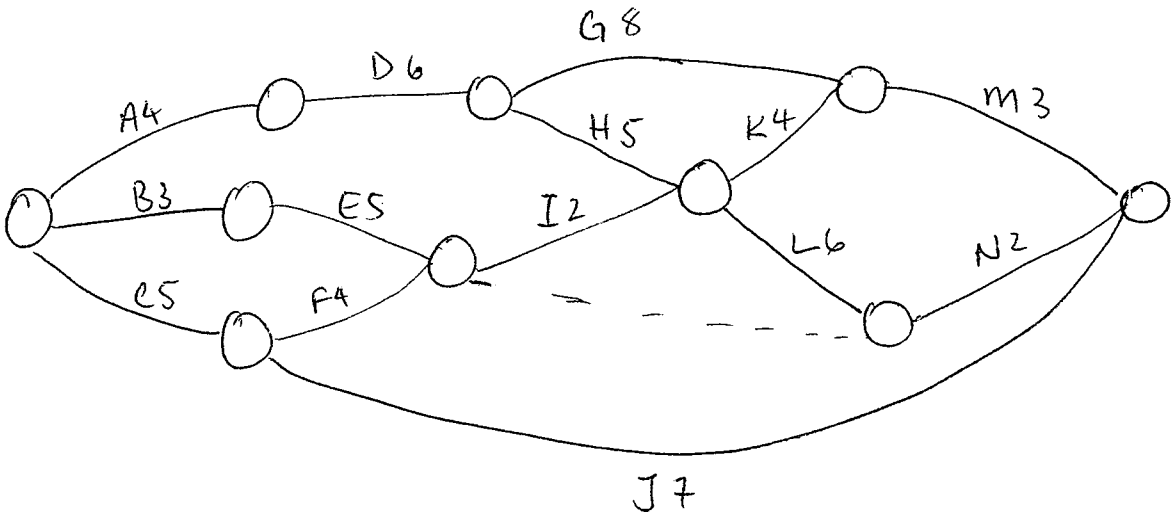
Task Time

Task	A	B	C	D	E	F	G	H	I	J	K	L	M	N
Time (days)	4	3	5	6	5	4	8	5	2	7	4	6	3	2

Task Order

- Tasks A, B and C have no predecessors.
- Task D has A as a predecessor.
- Task E has B as a predecessor.
- Task F has C as a predecessor.
- Task D is the predecessor for tasks G and H.
- Tasks E and F are the predecessors for tasks I and J.
- Tasks H and I are the predecessors for tasks K and L.
- Task M has tasks G and K as predecessors.
- Task N has E, F and L as a predecessor.

Construct a project network below to clearly represent the given information.



7. (7 marks)

A dispatcher for the Emu Taxi Company has six taxis at different locations and five customers who have called for service. The distance from each taxi's current location to each customer is shown in the following table.

	Customer				
Task	**1**	**2**	**3**	**4**	**5**
A	7	2	4	10	7
B	5	1	5	6	6
C	8	7	6	5	5
D	2	5	2	4	5
E	3	3	5	8	4
F	6	2	4	3	4

By use of the Hungarian algorithm determine the optimal assignment of taxi's and customers that will minimise the distance travelled.

(1)
$$\begin{bmatrix} 7 & 2 & 4 & 10 & 7 & 0 \\ 5 & 1 & 5 & 6 & 6 & 0 \\ 8 & 7 & 6 & 5 & 5 & 0 \\ 2 & 5 & 2 & 4 & 5 & 0 \\ 3 & 3 & 5 & 8 & 4 & 0 \\ 6 & 2 & 4 & 3 & 4 & 0 \end{bmatrix}$$ ✓

(2) N/A

(3)
(4)
$$\begin{bmatrix} 5 & 1 & 2 & 7 & 3 & 0 \\ 3 & 0 & 3 & 3 & 2 & 0 \\ 6 & 6 & 4 & 2 & 1 & 0 \\ 0 & 4 & 0 & 1 & 1 & 0 \\ 1 & 2 & 3 & 5 & 0 & 0 \\ 4 & 1 & 2 & 0 & 0 & 0 \end{bmatrix}$$ ✓✓

$n \neq 6$

(5)
(4)
$$\begin{bmatrix} 4 & 0 & 1 & 6 & 2 & 0 \\ 3 & 0 & 3 & 3 & 2 & 1 \\ 5 & 5 & 3 & 1 & 0 & 0 \\ 0 & 4 & 0 & 1 & 1 & 1 \\ 1 & 2 & 3 & 5 & 0 & 1 \\ 4 & 1 & 2 & 0 & 0 & 1 \end{bmatrix}$$ ✓

$n \neq 6$

(5)
$$\begin{bmatrix} 3 & 0 & \boxed{0} & 5 & 2 & \textcircled{0} \\ 2 & \textcircled{0} & 2 & 2 & 2 & 1 \\ 4 & 5 & 2 & 0 & \textcircled{0} & \boxed{0} \\ \boxed{0} & 5 & \textcircled{0} & 1 & 2 & 2 \\ \textcircled{0} & 2 & 2 & 5 & \boxed{0} & 1 \\ 4 & 2 & 2 & \boxed{0} & 1 & 2 \end{bmatrix}$$ ✓

$n = 6$

ANS 1		ANS 2	
A → 3	: 4	A → N/A	: –
B → 2	: 1	B → 2	: 1
C → N/A	: –	C → 5	: 5
D → 1	: 2	D → 3	: 2
E → 5	: 4	E → 1	: 3
F → 4	: 3	F → 4	: 3
14 hrs		14 hrs	

✓✓ Allocation + Total

END OF QUESTIONS